# Graph

A graph is a mathematical structure that models the relationship between pairs of objects.

## A simple (weighted) graph



* The circle is called a **Vertices** or **node**
* The line that connects them is called an **edge** or an **arc**
* The arcs have no arrow (bidirectional) so graph is called an **undirected** graph
* Because the only edge is labelled (with a distance) the graph is called a **weighted** graph

### Question

Using the descriptions above, describe the following graph:



This is a ........................... , .............................. graph

### Uses of graphs

* Model complex real-life problems e.g
	+ Human networks
	+ Transport networks (roads/rail and rail fares)
	+ The internet
	+ Computer networks

### Implementing a graph (2 implmentations – adjacency matrix vs adjacency list )

#### The adjacency matrix

A 2D array can be used to store information about a directed or undirected graph. Each of the rows and columns represents a node and a value stored in the cell at the intersection of for i column j indicated that there is an edge connecting node i and node j.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | A | B | C | D | E | F |
| A |  | 5 | 4 |  |  |  |
| B |  |  | 6 | 3 |  |  |
| C |  |  |  |  |  | 8 |
| D |  |  |  |  | 2 |  |
| E |  |  |  |  |  |  |
| F |  |  |  |  |  |  |



In the case of an undirected graph, the adjacency matrix will be symmetric, with the same entry in row 0 column 1 as in row 1 column 0. For example, an unweighted graph may be represented with 1s instead of weights, in the relevant cells.

### Question

Draw an adjacency matrix to represent the weighted graph below:



|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | **Bury St Edmunds** | **Framlingham** | **Wickham Market** | **Wood-bridge** | **Ipswich** | **Stow-market** |
| **Bury St Edmunds** |  |  |  |  |  |  |
| **Framlingham** |  |  |  |  |  |  |
| **Wickham Market** |  |  |  |  |  |  |
| **Woodbridge** |  |  |  |  |  |  |
| **Ipswich** |  |  |  |  |  |  |
| **Stowmarket** |  |  |  |  |  |  |

**Advantages**: Convenient to work with / quick to add an arc / quick to test for presence of arc

**Disadvantages**: A sparse graph with many nodes but not many arcs will leave most of the cells empty. Large the graph the more memory space will be wasted.

#### The adjacency list

A list of all the nodes is created and each node points to a list of all the adjacent nodes to which it is directly linked. The adjacent list can be implemented as a list of dictionaries, with the key in each dictionary being the node and the value, the edge weight.

The graph in the (adjacency matrix) above would be represented as follows:

|  |  |  |  |
| --- | --- | --- | --- |
| A |  | → | {B:5, C:4} |
| B |  | → | {C:6, D:3} |
| C |  | → | {F:8} |
| D |  | → | {E:2} |
| E |  | → | {} |
| F |  | → | {} |

Advantages: more space efficient. Uses much less memory to represent a sparsely connected graph.

The unweighted version of the same graph would be represented as below (notice dictionary data is not required as there a no edge weights)

|  |  |  |  |
| --- | --- | --- | --- |
| A |  | → | [B,C] |
| B |  | → | [C,D] |
| C |  | → | [F] |
| D |  | → | [E] |
| E |  | → | [] |
| F |  | → | [] |

**Advantage**: uses much less memory

### Question

Draw an adjacency list to represent the unweighted graph show below (but assuming this time that it is undirected)

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |



### Question

The figure below shows an adjacency matrix representation of a directed graph (digraph)

|  |  |  |
| --- | --- | --- |
|  |  | **To** |
|  |  | A | B | C | D | E |
| **From** | A | 0 | 5 | 3 | 10 | 0 |
| B | 0 | 0 | 1 | 8 | 0 |
| C | 0 | 0 | 0 | 7 | 6 |
| D | 0 | 0 | 0 | 0 | 4 |
| E | 0 | 0 | 0 | 0 | 0 |

**Question**: Draw a diagram of the directed graph, showing edge weights

**Question**: Draw an adjacency list representing this graph

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

**Question**: Give one advantage of using an adjacency matrix to represent a graph and one advantage of using an adjacency list. Explain the circumstances in which each is more appropriate.

**Question:** Graph algorithms are used with GPS nav systems, social networking, gaming etc. Describe two practical applications of graphs. (help: Google“practical applications of graph algorithms”)