Chapter 51 – Sets

Objectives

- Be familiar with the concept of a set and the notations used for specifying a set and set comprehension
- Be familiar with the compact representation of a set
- Be familiar with the concept of finite and infinite sets, countably infinite sets, cardinality of a finite set, Cartesian product of sets
- Be familiar with the meaning of the terms subset, proper subset, countable set
- Be familiar with set operations: membership, union, intersection, difference

Definition of a set

A **set** is an unordered collection of values or symbols in which each value or symbol occurs at most once.

A set may be defined in one of three ways, and the notation used for each of these is explained below.

Defining a set by listing each member

The list of members is enclosed in curly brackets:

e.g. A = {2, 4, 6, 8}

Q1: Define a set A consisting of all prime numbers between 1 and 20.

Common sets

There are some sets that are used so often that they have special names and notational conventions to identify them. These include:

- The empty set {} or Ø, which has no elements.
- The (infinite) set of natural numbers, including zero, referred to as $\mathbb{N}$ in mathematics.

  N or $\mathbb{N}$ = {0, 1, 2, 3, ...}

A **natural** number is a whole number that is used in counting. For example, five gold rings, four calling birds, three French hens. (This is sometimes defined as {1, 2, 3, ...} without including zero.)

Note that the ellipsis ("...") indicates that the set continues in the obvious way, and can be used to indicate an infinite set.

- The set of all **integers** whether positive, negative or zero:

  Z or $\mathbb{Z}$ = {..., -2, -1, 0, 1, 2, ...}

- The set of all **rational numbers** **Q** or $\mathbb{Q}$, i.e. any value that can be expressed as a ratio, or fraction. This includes all integer values since each can simply be expressed as 7/1 or 1076/1, to use the examples above.
- The set **R** or $\mathbb{R}$ of **real numbers** is defined as 'the set of all possible real world quantities'. This includes, for example, -10, -6.456, 0 4, 6.0, √2 and π. It does not include 'imaginary' numbers such as √-1, or infinity (∞).

Q2: Define a set B of all positive integers divisible by 2.

Finite and infinite sets

A **finite set** is one whose elements can be counted off by natural numbers up to a particular number. For example, 10 is the fourth and final element of the set A = {1, 4, 6, 10}.

Another example of a finite set is the set of all odd numbers from 1 to 99, which may be specified as:

A = {1, 3, 5, ..., 99}

Again, the ellipsis (...) indicates that the list continues in the obvious way.

The **cardinality** of a finite set is the number of elements in the set.

An **infinite set** may be countable or uncountable. For example, $\mathbb{N}$ (the set of natural numbers) and $\mathbb{R}$ (the set of real numbers) are examples of infinite sets, because they cannot be counted off against the set of natural numbers up to a certain number.

$\mathbb{N}$ is a **countably infinite set** because you can count the elements off against the set of natural numbers; 0, 1, 2, 3 and so on. This is in contrast to the set $\mathbb{R}$ which is not countable; you cannot list all the numbers in the set or say which is the next number.

A **countable** set is a set which can be counted off against a subset of the natural numbers, i.e. all of the natural numbers up to a fixed limit. A countably infinite set is one which can be counted off against the natural numbers but without ever stopping.

Defining a set by set comprehension

A set may be defined by **set comprehension**, using the notation shown in the example below:

$B = \{n^2 \mid n \in \mathbb{N} \wedge n < 5\}$

- The vertical bar | means "such that"
- The $\in$ symbol indicates membership, so $x \in \mathbb{N}$ is read as "x belongs to $\mathbb{N}$"
- $\wedge$ means "and"

Another way of writing the set B, therefore, is

B = {0, 1, 4, 9, 16}

Q3: Given that $A = \{x \mid x \in \mathbb{N} \wedge x \geq 1\}$, complete the sentence "A is the set consisting of those elements *x* such that ..."

Q4: Define set A = {0, 1, 8, 27, 64} using set comprehension.

Q5: List the numbers in the following set: $A = \{\{2x \mid x \in \mathbb{N} \wedge x \geq 1 \wedge x \leq 4\}\}$

Defining a set using the compact representation

A set may be defined using the **compact representation**, as in the following example:

$A = \{0^n1^n|n\}$

In this notation, A is the set containing all strings with an equal number of 0s and 1s.

Another way of writing this set is A = {01, 0011, 000111, 00001111, ...}

Q6: Using set comprehension or compact representation, define:

(a) the set A consisting of all the positive integers divisible by 5.

(b) the set B consisting of all positive integers between 1 and 9.

(c) the set C consisting of all positive integer powers of 2.

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Sets

Please read through p.265 onwards and complete this worksheet
http://www.rapidtables.com/math/symbols/Set_Symbols.htm

A set is an unordered collection of values in which each value or symbol occurs at most once.

An empty set can be written as?

An infinite set of natural numbers can be written as = {0, 1, 2, 4 ...}

The in the example above indicates that the set continues in some obvious way and can be used to indicate an infinite set

The set that is used to define **real numbers** is ....an example is: R = {-10, -20, -30}

The set that is used to define **rational numbers** is ....an example is: Q = {1/2, 1/3, ¼}

The set that is used to define **integars** is ....an example is: Z = {-2, -1, 0, 1, 2, ...}

Question

Define a set A consisting of all prime numbers between 1 and 20

Answer

Question

Define a set B of all positive integers divisible by 2

Answer

Defining a set by set comprehension

| means "such that"
indicates membership (element of) (example: x N means x belongs to N)
means "and" (the sign is sometimes called a **logical conjunction**)

Question

Given that A = {x | x N x≥ 1}, complete the sentence
"A is the set consisting of those elements x such that...

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Question

Define set A = {0, 1, 8, 27, 64} using set comprehension

Question

List the numbers in the following set: A = {{2x | x N $x \geq 1$ $x \leq 4$}},

Question

Using set comprehension or compact representation, define:

(a) The set A consisting of all positive integers divisible by 5

(b) The set B consisting of all positive integers between 1 and 9

(c) The set C consisting of all positive integers powers of 2

Cartesian product of two sets

The Cartesian product of two sets A and B, written A xB and spoken out loud as “A cross B”, is the set of all ordered pairs (a,b) where a is a member of A and b is a member of B.

Example, the set A is defined as A = {1, 3, 5} and the set B as B = {12, 25, 40} The definition of C, which is defined as A x B is written:

C = {(1, 12), (1,25), (1,40), (3,12), (3,25), (3,40),(5,12),(5,25),(5,40)}

Question

What is the Cartesian product of sets S1 and S2, where S1 = {4, 8, 3} and S2 = {8}?

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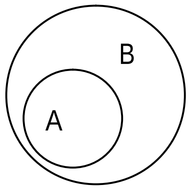
Subsets



If every member of Set A is also a member of set B, then A is a subset of B written as

$A \subseteq B$

An equivalent statement is “B is a superset of A” or “B contains A”, written as:

$B \supseteq A$

If A is a subset of, but not equal to B, then A is called a **proper subset** of B.

$A \subset B$

e.g. $\{0, 1, 2\} \subset N$

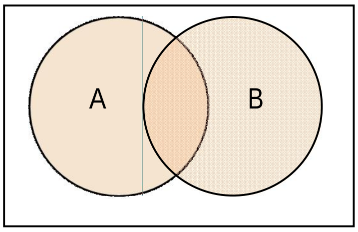
Question

If A is the set of prime numbers less than 10, B is the set of odd numbers less that 10 and C is the set of even numbers less than 10, which of the following statements are true?

$A \subseteq B$ $\quad B \subseteq A$ $\quad A \subseteq C$ $\quad C \subseteq A$ $\quad B \subseteq C$ $\quad C \subseteq B$

Union

Two sets A and B can be “added together”, resulting in the set that contains everything in either A or B. The **union** of A and B is denoted by



$A \cup B$

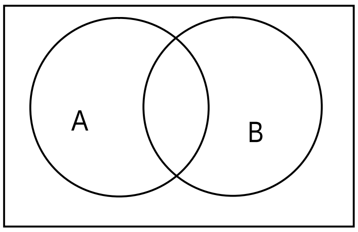
Examples: $A = \{1, 3, 5\}$ $B = \{3, 4, 8\}$

$A \cup B = \{1, 3, 4, 5, 8\}$

Question

If $A = \{1, 3, 5\}$, what is in the set $A \cup A$?

Intersection



The intersection of two sets contains all the members that both sets have in common. Thus the intersection of the two sets:
$A = \{1, 2, 3, 4, 5\}$ and $B = \{1, 3, 5, 7, 9\}$ is the set $\{1, 3, 5\}$
This is written as $A \cap B = \{1, 3, 5\}$

Question

If A is defined as $\{1, 2, 3\}$ and B as $\{2, 3, 4\}$, define the set as $A \cap B$

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Difference

The difference of two sets is denoted by A \ B.

If A = {1, 2, 3, 4} and B = {1, 3} then A \ B = {2, 4}

If A = {1, 2, 3, 4} and B = {1, 3, 5}, then A \ B = {2, 4}. "Subtracting" a member that is not in set A has no effect

Question

If A is the set of prime numbers less than 10 and B is the set of odd numbers less than 12, what numbers are in set C = B \ A?

Questions

There are two sets A = {A, E, I, O, U} and B = {H, E, L, O}

- Define the union of the two sets
- Define the Cartesian product of the two sets
- Define the intersection of the two sets
- Define the difference of the two sets
- Define cardinality
- Using set comprehension, define a set that contains integers that are less than or equal to 4